GLOBAL EDITION

College Mathematics

for Business, Economics, Life Sciences, and Social Sciences

THIRTEENTH EDITION

Raymond A. Barnett • Michael R. Ziegler • Karl E. Byleen

ALWAYS LEARNING

PEARSON

COLLEGE MATHEMATICS

FOR BUSINESS, ECONOMICS, LIFE SCIENCES, AND SOCIAL SCIENCES

Thirteenth Edition

Global Edition

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PREFACE

The thirteenth edition of *College Mathematics for Business, Economics, Life Sciences, and Social Sciences* is designed for a two-term (or condensed one-term) course in finite mathematics and calculus for students who have had one to two years of high school algebra or the equivalent. The book's overall approach, refined by the authors' experience with large sections of college freshmen, addresses the challenges of teaching and learning when prerequisite knowledge varies greatly from student to student.

The authors had three main goals when writing this text:

- To write a text that students can easily comprehend
- To make connections between what students are learning and how they may apply that knowledge
- To give flexibility to instructors to tailor a course to the needs of their students.

Many elements play a role in determining a book's effectiveness for students. Not only is it critical that the text be accurate and readable, but also, in order for a book to be effective, aspects such as the page design, the interactive nature of the presentation, and the ability to support and challenge all students have an incredible impact on how easily students comprehend the material. Here are some of the ways this text addresses the needs of students at all levels:

- Page layout is clean and free of potentially distracting elements.
- *Matched Problems* that accompany each of the completely worked examples help students gain solid knowledge of the basic topics and assess their own level of understanding before moving on.
- Review material (Appendix A and Chapters 1 and 2) can be used judiciously to help remedy gaps in prerequisite knowledge.
- A *Diagnostic Prerequisite Test* prior to Chapter 1 helps students assess their skills, while the *Basic Algebra Review* in Appendix A provides students with the content they need to remediate those skills.
- *Explore and Discuss* problems lead the discussion into new concepts or build upon a current topic. They help students of all levels gain better insight into the mathematical concepts through thought-provoking questions that are effective in both small and large classroom settings.
- Instructors are able to easily craft homework assignments that best meet the needs of their students by taking advantage of the variety of types and difficulty levels of the exercises. Exercise sets at the end of each section consist of a ***Skills Warm-up*** (four to eight problems that review prerequisite knowledge specific to that section) followed by problems of varying levels of difficulty.
- The MyMathLab course for this text is designed to help students help themselves and provide instructors with actionable information about their progress. The immediate feedback students receive when doing homework and practice in MyMathLab is invaluable, and the easily accessible e-book enhances student learning in a way that the printed page sometimes cannot.

Most important, all students get substantial experience in modeling and solving real-world problems through application examples and exercises chosen from business and economics, life sciences, and social sciences. Great care has been taken to write a book that is mathematically correct, with its emphasis on computational skills, ideas, and problem solving rather than mathematical theory.

Finally, the choice and independence of topics make the text readily adaptable to a variety of courses (see the chapter dependencies chart on page 13). This text is one of three books in the authors' college mathematics series. The others are *Finite Mathematics for Business, Economics, Life Sciences, and Social Sciences*, and *Calculus for Business, Economics, Life Sciences, and Social Sciences*. *Additional Calculus Topics*, a supplement written to accompany the Barnett/Ziegler/Byleen series, can be used in conjunction with any of these books.

New to This Edition

Fundamental to a book's effectiveness is classroom use and feedback. Now in its thirteenth edition, *College Mathematics for Business, Economics, Life Sciences, and Social Sciences* has had the benefit of a substantial amount of both. Improvements in this edition evolved out of the generous response from a large number of users of the last and previous editions as well as survey results from instructors, mathematics departments, course outlines, and college catalogs. In this edition,

- The Diagnostic Prerequisite Test has been revised to identify the specific deficiencies in prerequisite knowledge that cause students the most difficulty with finite mathematics and calculus.
- Most exercise sets now begin with a ***Skills Warm-up***—four to eight problems that review prerequisite knowledge specific to that section in a just-in-time approach. References to review material are given for the benefit of students who struggle with the warm-up problems and need a refresher.
- Section 6.1 has been rewritten to better motivate and introduce the simplex method and associated terminology.
- Section 14.4 has been rewritten to cover the trapezoidal rule and Simpson's rule.
- Examples and exercises have been given up-to-date contexts and data.
- Exposition has been simplified and clarified throughout the book.
- *MyMathLab* for this text has been enhanced greatly in this revision. Most notably, a "Getting Ready for Chapter X" has been added to each chapter as an optional resource for instructors and students as a way to address the prerequisite skills that students need, and are often missing, for each chapter. Many more improvements have been made. See the detailed description on pages 17 and 18 for more information.

Trusted Features

Emphasis and Style

As was stated earlier, this text is written for student comprehension. To that end, the focus has been on making the book both mathematically correct and accessible to students. Most derivations and proofs are omitted, except where their inclusion adds significant insight into a particular concept as the emphasis is on computational skills, ideas, and problem solving rather than mathematical theory. General concepts and results are typically presented only after particular cases have been discussed.

Design

One of the hallmark features of this text is the **clean, straightforward design** of its pages. Navigation is made simple with an obvious hierarchy of key topics and a judicious use of call-outs and pedagogical features. We made the decision to maintain a two-color design to

help students stay focused on the mathematics and applications. Whether students start in the chapter opener or in the exercise sets, they can easily reference the content, examples, and *Conceptual Insights* they need to understand the topic at hand. Finally, a functional use of color improves the clarity of many illustrations, graphs, and explanations, and guides students through critical steps (see pages 81, 128, and 422).

Examples and Matched Problems

More than 490 completely worked examples are used to introduce concepts and to demonstrate problem-solving techniques. Many examples have multiple parts, significantly increasing the total number of worked examples. The examples are annotated using blue text to the right of each step, and the problem-solving steps are clearly identified. **To give students extra help** in working through examples, dashed boxes are used to enclose steps that are usually performed mentally and rarely mentioned in other books (see Example 2 on page 24). Though some students may not need these additional steps, many will appreciate the fact that the authors do not assume too much in the way of prior knowledge.

EXAMPLE 9 Solving Exponential Equations Solve for x to four decimal places:

(A) $10^x = 2$ (B) $e^x = 3$ (C) $3^x = 4$

SOLUTION

(A)
$$10^x = 2$$ Take common logarithms of both sides.
$$\log 10^x = \log 2$$ Property 3
$$x = \log 2$$ Use a calculator.
$$= 0.3010$$ To four decimal places

(B)
$$e^x = 3$$ Take natural logarithms of both sides.
$$\ln e^x = \ln 3$$ Property 3
$$x = \ln 3$$ Use a calculator.
$$= 1.0986$$ To four decimal places

(C)
$$3^x = 4$$ Take either natural or common logarithms of both sides. (We choose common logarithms.)
$$\log 3^x = \log 4$$ Property 7
$$x \log 3 = \log 4$$ Solve for x.
$$x = \frac{\log 4}{\log 3}$$ Use a calculator.
$$= 1.2619$$ To four decimal places

Matched Problem 9 Solve for x to four decimal places:

(A) $10^x = 7$ (B) $e^x = 6$ (C) $4^x = 5$

Each example is followed by a similar *Matched Problem* for the student to work while reading the material. This actively involves the student in the learning process. The answers to these matched problems are included at the end of each section for easy reference.

Explore and Discuss

Most every section contains *Explore and Discuss* problems at appropriate places to encourage students to think about a relationship or process before a result is stated or to investigate additional consequences of a development in the text. This serves to foster critical thinking and communication skills. The Explore and Discuss material can be used for in-class discussions or out-of-class group activities and is effective in both small and large class settings.

Explore and Discuss 2 How many x intercepts can the graph of a quadratic function have? How many y intercepts? Explain your reasoning.

Exercise Sets

The book contains over 6,500 carefully selected and graded exercises. Many problems have multiple parts, significantly increasing the total number of exercises. Exercises are paired so that consecutive odd- and even-numbered exercises are of the same type and difficulty level. Each exercise set is designed to allow instructors to craft just the right assignment for students. The *writing exercises*, indicated by the icon ✎, provide students with an opportunity to express their understanding of the topic in writing. Answers to all odd-numbered problems are in the back of the book. Answers to application problems in linear programming include both the mathematical model and the numeric answer.

Applications

A major objective of this book is to give the student substantial experience in modeling and solving real-world problems. Enough applications are included to convince even the most skeptical student that mathematics is really useful (see the Index of Applications at the back of the book). Almost every exercise set contains application problems, including applications from business and economics, life sciences, and social sciences. An instructor with students from all three disciplines can let them choose applications from their own field of interest; if most students are from one of the three areas, then special emphasis can be placed there. Most of the applications are simplified versions of actual real-world problems inspired by professional journals and books. No specialized experience is required to solve any of the application problems.

Additional Pedagogical Features

The following features, while helpful to any student, are particularly helpful to students enrolled in a large classroom setting where access to the instructor is more challenging or just less frequent. These features provide much-needed guidance for students as they tackle difficult concepts.

- **Call-out boxes** highlight important definitions, results, and step-by-step processes (see pages 110, 116–117).
- **Caution statements** appear throughout the text where student errors often occur (see pages 158, 163, and 196).

⚠ **CAUTION** Note that in Example 11 we let $x = 0$ represent 1900. If we let $x = 0$ represent 1940, for example, we would obtain a different logarithmic regression equation, but the prediction for 2015 would be the same. We would *not* let $x = 0$ represent 1950 (the first year in Table 1) or any later year, because logarithmic functions are undefined at 0. ▲

- **Conceptual Insights**, appearing in nearly every section, often make explicit connections to previous knowledge, but sometimes encourage students to think beyond the particular skill they are working on and see a more enlightened view of the concepts at hand (see pages 79, 160, 236).

CONCEPTUAL INSIGHT

The notation (2.7) has two common mathematical interpretations: the ordered pair with first coordinate 2 and second coordinate 7, and the open interval consisting of all real numbers between 2 and 7. The choice of interpretation is usually determined by the context in which the notation is used. The notation $(2, -7)$ could be interpreted as an ordered pair but not as an interval. In interval notation, the left endpoint is always written first. So, $(-7, 2)$ is correct interval notation, but $(2, -7)$ is not.

- The newly revised **Diagnostic Prerequisite Test**, located at the front of the book, provides students with a tool to assess their prerequisite skills prior to taking the course. The **Basic Algebra Review**, in Appendix A, provides students with seven sections of content to help them remediate in specific areas of need. Answers to the Diagnostic Prerequisite Test are at the back of the book and reference specific sections in the Basic Algebra Review or Chapter 1 for students to use for remediation.

Graphing Calculator and Spreadsheet Technology

Although access to a graphing calculator or spreadsheets is not assumed, it is likely that many students will want to make use of this technology. To assist these students, optional graphing calculator and spreadsheet activities are included in appropriate places. These include brief discussions in the text, examples or portions of examples solved on a graphing calculator or spreadsheet, and exercises for the student to solve. For example, linear regression is introduced in Section 1.3, and regression techniques on a graphing calculator are used at appropriate points to illustrate mathematical modeling with real data. All the optional graphing calculator material is clearly identified with the icon and can be omitted without loss of continuity, if desired. Optional spreadsheet material is identified with the icon. Graphing calculator screens displayed in the text are actual output from the TI-84 Plus graphing calculator.

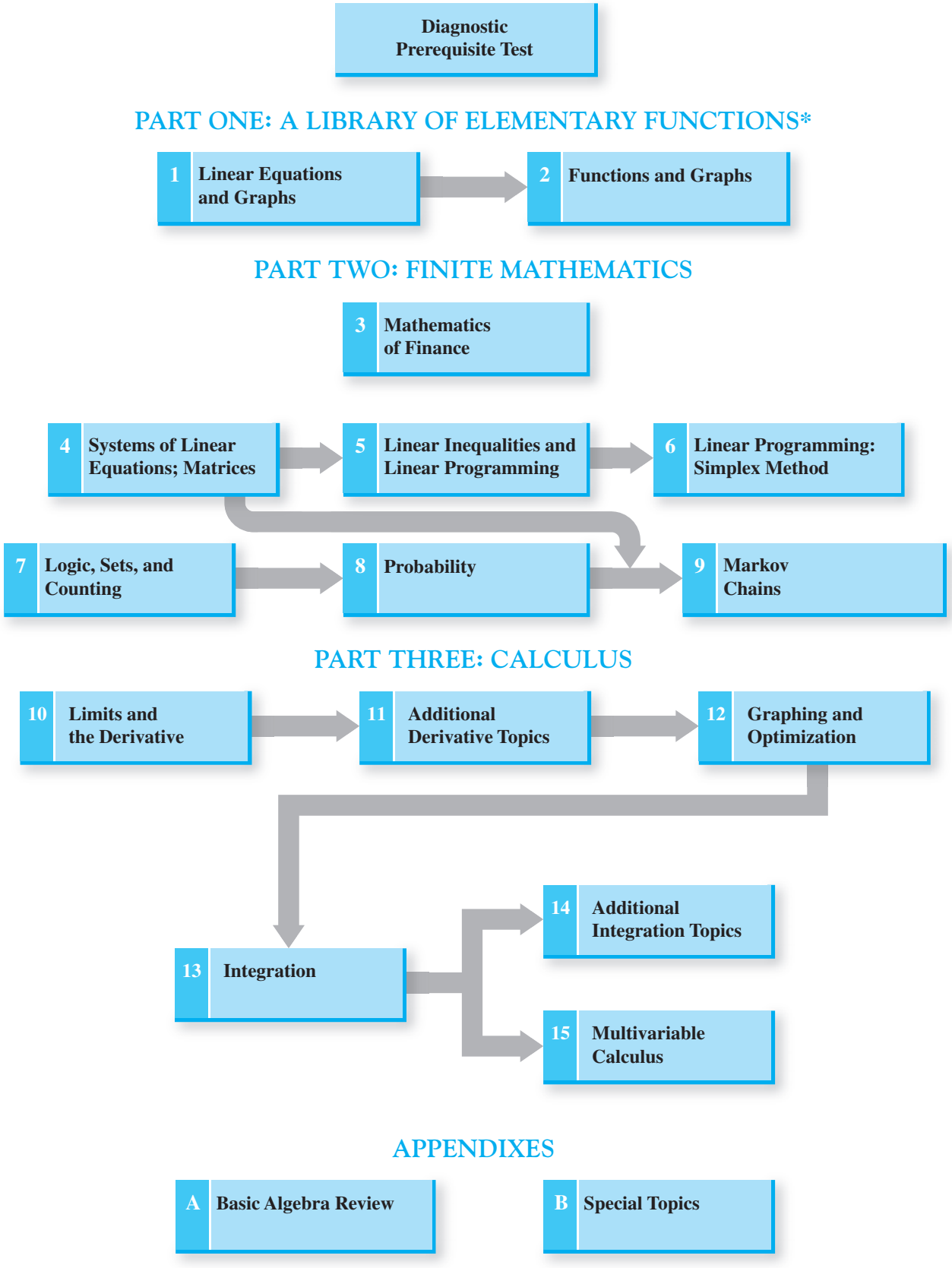
Chapter Reviews

Often it is during the preparation for a chapter exam that concepts gel for students, making the chapter review material particularly important. The chapter review sections in this text include a comprehensive summary of important terms, symbols, and concepts, keyed to completely worked examples, followed by a comprehensive set of Review Exercises. Answers to Review Exercises are included at the back of the book; *each answer contains a reference to the section in which that type of problem is discussed* so students can remediate any deficiencies in their skills on their own.

Content

The text begins with the development of a library of elementary functions in **Chapters 1 and 2**, including their properties and applications. Many students will be familiar with most, if not all, of the material in these introductory chapters. Depending on students'

Chapter Dependencies



*Selected topics from Part One may be referred to as needed in Parts Two or Three or reviewed systematically before starting Part Two.

preparation and the course syllabus, an instructor has several options for using the first two chapters, including the following:

(i) Skip Chapters 1 and 2 and refer to them only as necessary later in the course;

(ii) Cover Chapter 1 quickly in the first week of the course, emphasizing price–demand equations, price–supply equations, and linear regression, but skip Chapter 2;

(iii) Cover Chapters 1 and 2 systematically before moving on to other chapters.

The material in Part Two (Finite Mathematics) can be thought of as four units:

1. Mathematics of finance (Chapter 3)
2. Linear algebra, including matrices, linear systems, and linear programming (Chapters 4, 5, and 6)
3. Probability and statistics (Chapters 7 and 8)
4. Applications of linear algebra and probability to Markov chains (Chapter 9)

The first three units are independent of each other, while the fourth unit is dependent on some of the earlier chapters (see chart on previous page).

- **Chapter 3** presents a thorough treatment of simple and compound interest and present and future value of ordinary annuities. Appendix B.1 addresses arithmetic and geometric sequences and can be covered in conjunction with this chapter, if desired.
- **Chapter 4** covers linear systems and matrices with an emphasis on using row operations and Gauss–Jordan elimination to solve systems and to find matrix inverses. This chapter also contains numerous applications of mathematical modeling using systems and matrices. To assist students in formulating solutions, all answers at the back of the book for application exercises in Sections 4.3, 4.5, and the chapter Review Exercises contain both the mathematical model and its solution. The row operations discussed in Sections 4.2 and 4.3 are required for the simplex method in Chapter 6. Matrix multiplication, matrix inverses, and systems of equations are required for Markov chains in Chapter 9.
- **Chapters 5 and 6** provide a broad and flexible coverage of linear programming. Chapter 5 covers two-variable graphing techniques. Instructors who wish to emphasize linear programming techniques can cover the basic simplex method in Sections 6.1 and 6.2 and then discuss either or both of the following: the dual method (Section 6.3) and the big M method (Section 6.4). Those who want to emphasize modeling can discuss the formation of the mathematical model for any of the application examples in Sections 6.2–6.4, and either omit the solution or use software to find the solution. To facilitate this approach, all answers at the back of the book for application exercises in Sections 6.2–6.4 and the chapter Review Exercises contain both the mathematical model and its solution.
- **Chapter 7** provides a foundation for probability with a treatment of logic, sets, and counting techniques.
- **Chapter 8** covers basic probability, including Bayes' formula and random variables.
- **Chapter 9** ties together concepts developed in earlier chapters and applies them to Markov chains. This provides an excellent unifying conclusion to a finite mathematics course.

The material in Part Three (Calculus) consists of differential calculus (Chapters 10–12), integral calculus (Chapters 13 and 14), multivariable calculus (Chapter 15). In general, Chapters 10–12 must be covered in sequence; however, certain sections can be omitted or given brief treatments, as pointed out in the discussion that follows (see the Chapter Dependencies chart on page 13).

- **Chapter 10** introduces the derivative. The first three sections cover limits (including infinite limits and limits at infinity), continuity, and the limit properties that are essential to understanding the definition of the derivative in Section 10.4. The remaining sections of the chapter cover basic rules of differentiation, differentials, and applications of derivatives in business and economics. The interplay between graphical, numerical, and algebraic concepts is emphasized here and throughout the text.
- In **Chapter 11** the derivatives of exponential and logarithmic functions are obtained before the product rule, quotient rule, and chain rule are introduced. Implicit differentiation is introduced in Section 11.5 and applied to related rates problems in Section 11.6. Elasticity of demand is introduced in Section 11.7. The topics in these last three sections of Chapter 11 are not referred to elsewhere in the text and can be omitted.
- **Chapter 12** focuses on graphing and optimization. The first two sections cover first-derivative and section-derivative graph properties. L'Hôpital's rule is discussed in Section 12.3. A graphing strategy is presented and illustrated in Section 12.4. Optimization is covered in Sections 12.5 and 12.6, including examples and problems involving end-point solutions.
- **Chapter 13** introduces integration. The first two sections cover antidifferentiation techniques essential to the remainder of the text. Section 13.3 discusses some applications involving differential equations that can be omitted. The definite integral is defined in terms of Riemann sums in Section 13.4 and the fundamental theorem of calculus is discussed in Section 13.5. As before, the interplay between graphical, numerical, and algebraic properties is emphasized. These two sections are also required for the remaining chapters in the text.
- **Chapter 14** covers additional integration topics and is organized to provide maximum flexibility for the instructor. The first section extends the area concepts introduced in Chapter 14 to the area between two curves and related applications. Section 14.2 covers three more applications of integration, and Sections 14.3 and 14.4 deal with additional methods of integration, including integration by parts, the trapezoidal rule, and Simpson's rule. Any or all of the topics in Chapter 14 can be omitted.
- **Chapter 15** deals with multivariable calculus. The first five sections can be covered any time after Section 12.6 has been completed. Sections 15.6 and 15.7 require the integration concepts discussed in Chapter 13.
- **Appendix A** contains a concise review of basic algebra that may be covered as part of the course or referenced as needed. As mentioned previously, **Appendix B** contains additional topics that can be covered in conjunction with certain sections in the text, if desired.

Accuracy Check

Because of the careful checking and proofing by a number of mathematics instructors (acting independently), the authors and publisher believe this book to be substantially error free. If an error should be found, the authors would be grateful if notification were sent to Karl E. Byleen, 9322 W. Garden Court, Hales Corners, WI 53130; or by e-mail to kbyleen@wi.rr.com.

Student Supplements

Additional Calculus Topics to Accompany Calculus, 13e, and College Mathematics, 13e

- This separate book contains three unique chapters: Differential Equations, Taylor Polynomials and Infinite Series, and Probability and Calculus.
- ISBN 13: 978-0-321-93169-6; ISBN 10: 0-321-931696

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- Available in MyMathLab.

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- By Stela Pudar-Hozo, Indiana University–Northwest
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- By Salvatore Sciandra, Niagara County Community College
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Diagnostic Prerequisite Test

Work all of the problems in this self-test without using a calculator. Then check your work by consulting the answers in the back of the book. Where weaknesses show up, use the reference that follows each answer to find the section in the text that provides the necessary review.

1. Replace each question mark with an appropriate expression that will illustrate the use of the indicated real number property:

(A) Commutative $(\cdot)$: $x(y + z) = ?$

(B) Associative $(+)$: $2 + (x + y) = ?$

(C) Distributive: $(2 + 3)x = ?$

Problems 2–6 refer to the following polynomials:

(A) $3x - 4$ (B) $x + 2$

(C) $2 - 3x^2$ (D) $x^3 + 8$

2. Add all four.

3. Subtract the sum of (A) and (C) from the sum of (B) and (D).

4. Multiply (C) and (D).

5. What is the degree of each polynomial?

6. What is the leading coefficient of each polynomial?

In Problems 7 and 8, perform the indicated operations and simplify.

7. $5x^2 - 3x[4 - 3(x - 2)]$

8. $(2x + y)(3x - 4y)$

In Problems 9 and 10, factor completely.

9. $x^2 + 7x + 10$

10. $x^3 - 2x^2 - 15x$

11. Write 0.35 as a fraction reduced to lowest terms.

12. Write $\frac{7}{8}$ in decimal form.

13. Write in scientific notation:

(A) 4,065,000,000,000 (B) 0.0073

14. Write in standard decimal form:

(A) 2.55×10^8 (B) 4.06×10^{-4}

15. Indicate true (T) or false (F):

(A) A natural number is a rational number.

(B) A number with a repeating decimal expansion is an irrational number.

16. Give an example of an integer that is not a natural number.

In Problems 17–24, simplify and write answers using positive exponents only. All variables represent positive real numbers.

17. $6(xy^3)^5$

18. $\dfrac{9u^8v^6}{3u^4v^8}$

19. $(2 \times 10^5)(3 \times 10^{-3})$

20. $(x^{-3}y^2)^{-2}$

21. $u^{5/3}u^{2/3}$

22. $(9a^4b^{-2})^{1/2}$

23. $\dfrac{5^0}{3^2} + \dfrac{3^{-2}}{2^{-2}}$

24. $(x^{1/2} + y^{1/2})^2$

In Problems 25–30, perform the indicated operation and write the answer as a simple fraction reduced to lowest terms. All variables represent positive real numbers.

25. $\dfrac{a}{b} + \dfrac{b}{a}$

26. $\dfrac{a}{bc} - \dfrac{c}{ab}$

27. $\dfrac{x^2}{y} \cdot \dfrac{y^6}{x^3}$

28. $\dfrac{x}{y^3} \div \dfrac{x^2}{y}$

29. $\dfrac{\dfrac{1}{7+h} - \dfrac{1}{7}}{h}$

30. $\dfrac{x^{-1} + y^{-1}}{x^{-2} - y^{-2}}$

31. Each statement illustrates the use of one of the following real number properties or definitions. Indicate which one.

Commutative $(+, \cdot)$	Associative $(+, \cdot)$	Distributive
Identity $(+, \cdot)$	Inverse $(+, \cdot)$	Subtraction
Division	Negatives	Zero

(A) $(-7) - (-5) = (-7) + [-(-5)]$

(B) $5u + (3v + 2) = (3v + 2) + 5u$

(C) $(5m - 2)(2m + 3) = (5m - 2)2m + (5m - 2)3$

(D) $9 \cdot (4y) = (9 \cdot 4)y$

(E) $\dfrac{u}{-(v - w)} = \dfrac{u}{w - v}$

(F) $(x - y) + 0 = (x - y)$

32. Round to the nearest integer:

(A) $\frac{17}{3}$ (B) $-\frac{5}{19}$

33. Multiplying a number x by 4 gives the same result as subtracting 4 from x. Express as an equation, and solve for x.

34. Find the slope of the line that contains the points $(3, -5)$ and $(-4, 10)$.

35. Find the x and y coordinates of the point at which the graph of $y = 7x - 4$ intersects the x axis.

36. Find the x and y coordinates of the point at which the graph of $y = 7x - 4$ intersects the y axis.

In Problems 37 and 38, factor completely.

37. $x^2 - 3xy - 10y^2$

38. $6x^2 - 17xy + 5y^2$

In Problems 39–42, write in the form $ax^p + by^q$ where a, b, p, and q are rational numbers.

39. $\dfrac{3}{x} + 4\sqrt{y}$

40. $\dfrac{8}{x^2} - \dfrac{5}{y^4}$

41. $\dfrac{2}{5x^{3/4}} - \dfrac{7}{6y^{2/3}}$

42. $\dfrac{1}{3\sqrt{x}} + \dfrac{9}{\sqrt[3]{y}}$

In Problems 43 and 44, write in the form $a + b\sqrt{c}$ where a, b, and c are rational numbers.

43. $\dfrac{1}{4 - \sqrt{2}}$

44. $\dfrac{5 - \sqrt{3}}{5 + \sqrt{3}}$

In Problems 45–50, solve for x.

45. $x^2 = 5x$

46. $3x^2 - 21 = 0$

47. $x^2 - x - 20 = 0$

48. $-6x^2 + 7x - 1 = 0$

49. $x^2 + 2x - 1 = 0$

50. $x^4 - 6x^2 + 5 = 0$

PART 1

A LIBRARY OF ELEMENTARY FUNCTIONS

1 Linear Equations and Graphs

1.1 Linear Equations and Inequalities

1.2 Graphs and Lines

1.3 Linear Regression

Chapter 1 Summary and Review

Review Exercises

Introduction

We begin by discussing some algebraic methods for solving equations and inequalities. Next, we introduce coordinate systems that allow us to explore the relationship between algebra and geometry. Finally, we use this algebraic–geometric relationship to find equations that can be used to describe real-world data sets. For example, in Section 1.3 you will learn how to find the equation of a line that fits data on winning times in an Olympic swimming event (see Problems 27 and 28 on page 57). We also consider many applied problems that can be solved using the concepts discussed in this chapter.

1.1 Linear Equations and Inequalities

- Linear Equations
- Linear Inequalities
- Applications

The equation

$$3 - 2(x + 3) = \frac{x}{3} - 5$$

and the inequality

$$\frac{x}{2} + 2(3x - 1) \geq 5$$

are both first degree in one variable. In general, a **first-degree**, or **linear**, **equation** in one variable is any equation that can be written in the form

$$\textbf{Standard form: } ax + b = 0 \quad a \neq 0 \tag{1}$$

If the equality symbol, $=$, in (1) is replaced by $<$, $>$, $\leq$, or $\geq$, the resulting expression is called a **first-degree**, or **linear**, **inequality**.

A **solution** of an equation (or inequality) involving a single variable is a number that when substituted for the variable makes the equation (or inequality) true. The set of all solutions is called the **solution set**. When we say that we **solve an equation** (or inequality), we mean that we find its solution set.

Knowing what is meant by the solution set is one thing; finding it is another. We start by recalling the idea of equivalent equations and equivalent inequalities. If we perform an operation on an equation (or inequality) that produces another equation (or inequality) with the same solution set, then the two equations (or inequalities) are said to be **equivalent**. The basic idea in solving equations or inequalities is to perform operations that produce simpler equivalent equations or inequalities and to continue the process until we obtain an equation or inequality with an obvious solution.

Linear Equations

Linear equations are generally solved using the following equality properties.

THEOREM 1 Equality Properties

An equivalent equation will result if

1. The same quantity is added to or subtracted from each side of a given equation.
2. Each side of a given equation is multiplied by or divided by the same nonzero quantity.

EXAMPLE 1 Solving a Linear Equation Solve and check:

$$8x - 3(x - 4) = 3(x - 4) + 6$$

SOLUTION

$$\begin{aligned}
8x - 3(x - 4) &= 3(x - 4) + 6 && \text{Use the distributive property.}\\
8x - 3x + 12 &= 3x - 12 + 6 && \text{Combine like terms.}\\
5x + 12 &= 3x - 6 && \text{Subtract } 3x \text{ from both sides.}\\
2x + 12 &= -6 && \text{Subtract 12 from both sides.}\\
2x &= -18 && \text{Divide both sides by 2.}\\
x &= -9
\end{aligned}$$

CHECK

$$\begin{aligned}
8x - 3(x - 4) &= 3(x - 4) + 6\\
8(-9) - 3[(-9) - 4] &\stackrel{?}{=} 3[(-9) - 4] + 6\\
-72 - 3(-13) &\stackrel{?}{=} 3(-13) + 6\\
-33 &\stackrel{\checkmark}{=} -33
\end{aligned}$$

Matched Problem 1 Solve and check: $3x - 2(2x - 5) = 2(x + 3) - 8$

Explore and Discuss 1 According to equality property 2, multiplying both sides of an equation by a nonzero number always produces an equivalent equation. What is the smallest positive number that you could use to multiply both sides of the following equation to produce an equivalent equation without fractions?

$$\frac{x+1}{3} - \frac{x}{4} = \frac{1}{2}$$

EXAMPLE 2 Solving a Linear Equation Solve and check: $\frac{x+2}{2} - \frac{x}{3} = 5$

SOLUTION What operations can we perform on

$$\frac{x+2}{2} - \frac{x}{3} = 5$$

to eliminate the denominators? If we can find a number that is exactly divisible by each denominator, we can use the multiplication property of equality to clear the denominators. The LCD (least common denominator) of the fractions, 6, is exactly what we are looking for! Actually, any common denominator will do, but the LCD results in a simpler equivalent equation. So, we multiply both sides of the equation by 6:

$$6\left(\frac{x+2}{2} - \frac{x}{3}\right) = 6 \cdot 5 \quad *$$

$$\overset{3}{\cancel{6}} \cdot \frac{(x+2)}{\underset{1}{\cancel{2}}} - \overset{2}{\cancel{6}} \cdot \frac{x}{\underset{1}{\cancel{3}}} = 30$$

$$\begin{aligned} 3(x+2) - 2x &= 30 && \text{Use the distributive property.} \\ 3x + 6 - 2x &= 30 && \text{Combine like terms.} \\ x + 6 &= 30 && \text{Subtract 6 from both sides.} \\ x &= 24 \end{aligned}$$

CHECK

$$\begin{aligned} \frac{x+2}{2} - \frac{x}{3} &= 5 \\ \frac{24+2}{2} - \frac{24}{3} &\stackrel{?}{=} 5 \\ 13 - 8 &\stackrel{?}{=} 5 \\ 5 &\stackrel{\checkmark}{=} 5 \end{aligned}$$

Matched Problem 2 Solve and check: $\frac{x+1}{3} - \frac{x}{4} = \frac{1}{2}$

In many applications of algebra, formulas or equations must be changed to alternative equivalent forms. The following example is typical.

EXAMPLE 3 Solving a Formula for a Particular Variable If you deposit a principal P in an account that earns simple interest at an annual rate r, then the amount A in the account after t years is given by $A = P + Prt$. Solve for

(A) r in terms of A, P, and t

(B) P in terms of A, r, and t

*Dashed boxes are used throughout the book to denote steps that are usually performed mentally.

SOLUTION (A)

$$A = P + Prt \quad \text{Reverse equation.}$$
$$P + Prt = A \quad \text{Subtract } P \text{ from both sides.}$$
$$Prt = A - P \quad \text{Divide both members by } Pt.$$
$$r = \frac{A - P}{Pt}$$

(B)

$$A = P + Prt \quad \text{Reverse equation.}$$
$$P + Prt = A \quad \text{Factor out } P \text{ (note the use of the distributive property).}$$
$$P(1 + rt) = A \quad \text{Divide by } (1 + rt).$$
$$P = \frac{A}{1 + rt}$$

Matched Problem 3 If a cardboard box has length L, width W, and height H, then its surface area is given by the formula $S = 2LW + 2LH + 2WH$. Solve the formula for

(A) L in terms of S, W, and H (B) H in terms of S, L, and W

Linear Inequalities

Before we start solving linear inequalities, let us recall what we mean by $<$ (less than) and $>$ (greater than). If a and b are real numbers, we write

$$a < b \quad a \text{ is less than } b$$

if there exists a positive number p such that $a + p = b$. Certainly, we would expect that if a positive number was added to any real number, the sum would be larger than the original. That is essentially what the definition states. If $a < b$, we may also write

$$b > a \quad b \text{ is greater than } a.$$

EXAMPLE 4 Inequalities

(A) $3 < 5$ Since $3 + 2 = 5$

(B) $-6 < -2$ Since $-6 + 4 = -2$

(C) $0 > -10$ Since $-10 < 0$ (because $-10 + 10 = 0$)

Matched Problem 4 Replace each question mark with either $<$ or $>$.

(A) $2 \;?\; 8$ (B) $-20 \;?\; 0$ (C) $-3 \;?\; -30$

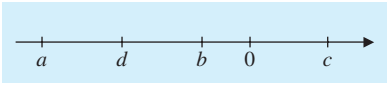


Figure 1 $a < b, c > d$

The inequality symbols have a very clear geometric interpretation on the real number line. If $a < b$, then a is to the left of b on the number line; if $c > d$, then c is to the right of d on the number line (Fig. 1). Check this geometric property with the inequalities in Example 4.

Explore and Discuss 2 Replace ? with $<$ or $>$ in each of the following:

(A) $-1 \;?\; 3$ and $2(-1) \;?\; 2(3)$

(B) $-1 \;?\; 3$ and $-2(-1) \;?\; -2(3)$

(C) $12 \;?\; -8$ and $\frac{12}{4} \;?\; \frac{-8}{4}$

(D) $12 \;?\; -8$ and $\frac{12}{-4} \;?\; \frac{-8}{-4}$

Based on these examples, describe the effect of multiplying both sides of an inequality by a number.

The procedures used to solve linear inequalities in one variable are almost the same as those used to solve linear equations in one variable, but with one important exception, as noted in item 3 of Theorem 2.

THEOREM 2 Inequality Properties

An equivalent inequality will result, and the **sense or direction will remain the same** if each side of the original inequality

1. has the same real number added to or subtracted from it.
2. is multiplied or divided by the same *positive* number.

An equivalent inequality will result, and the **sense or direction will reverse** if each side of the original inequality

3. is multiplied or divided by the same *negative* number.

Note: Multiplication by 0 and division by 0 are not permitted.

Therefore, we can perform essentially the same operations on inequalities that we perform on equations, with the exception that **the sense of the inequality reverses if we multiply or divide both sides by a negative number**. Otherwise, the sense of the inequality does not change. For example, if we start with the true statement

$$-3 > -7$$

and multiply both sides by 2, we obtain

$$-6 > -14$$

and the sense of the inequality stays the same. But if we multiply both sides of $-3 > -7$ by -2, the left side becomes 6 and the right side becomes 14, so we must write

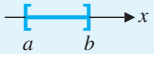
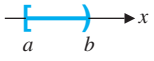
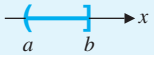
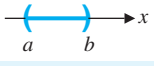
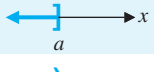
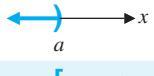
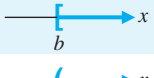
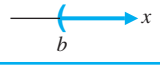
$$6 < 14$$

to have a true statement. The sense of the inequality reverses.

If $a < b$, the **double inequality** $a < x < b$ means that $\boldsymbol{a < x}$ **and** $\boldsymbol{x < b}$; that is, x is between a and b. **Interval notation** is also used to describe sets defined by inequalities, as shown in Table 1.

The numbers a and b in Table 1 are called the **endpoints** of the interval. An interval is **closed** if it contains all its endpoints and **open** if it does not contain any of its endpoints. The intervals $[a, b]$, $(-\infty, a]$, and $[b, \infty)$ are closed, and the intervals (a, b), $(-\infty, a)$,

Table 1 **Interval Notation**

Interval Notation	Inequality Notation	Line Graph
$[a, b]$	$a \le x \le b$	
$[a, b)$	$a \le x < b$	
$(a, b]$	$a < x \le b$	
(a, b)	$a < x < b$	
$(-\infty, a]$	$x \le a$	
$(-\infty, a)$	$x < a$	
$[b, \infty)$	$x \ge b$	
(b, ∞)	$x > b$	

and (b, ∞) are open. Note that the symbol ∞ (read infinity) is not a number. When we write $[b, \infty)$, we are simply referring to the interval that starts at b and continues indefinitely to the right. We never refer to ∞ as an endpoint, and we never write $[b, \infty]$. The interval $(-\infty, \infty)$ is the entire real number line.

Note that an endpoint of a line graph in Table 1 has a square bracket through it if the endpoint is included in the interval; a parenthesis through an endpoint indicates that it is not included.

CONCEPTUAL INSIGHT

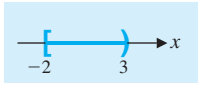
The notation $(2, 7)$ has two common mathematical interpretations: the ordered pair with first coordinate 2 and second coordinate 7, and the open interval consisting of all real numbers between 2 and 7. The choice of interpretation is usually determined by the context in which the notation is used. The notation $(2, -7)$ could be interpreted as an ordered pair but not as an interval. In interval notation, the left endpoint is always written first. So, $(-7, 2)$ is correct interval notation, but $(2, -7)$ is not.

EXAMPLE 5 Interval and Inequality Notation, and Line Graphs

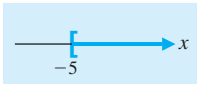
(A) Write $[-2, 3)$ as a double inequality and graph.

(B) Write $x \geq -5$ in interval notation and graph.

SOLUTION (A) $[-2, 3)$ is equivalent to $-2 \leq x < 3$.



(B) $x \geq -5$ is equivalent to $[-5, \infty)$.



Matched Problem 5

(A) Write $(-7, 4]$ as a double inequality and graph.

(B) Write $x < 3$ in interval notation and graph.

Explore and Discuss 3 The solution to Example 5B shows the graph of the inequality $x \geq -5$. What is the graph of $x < -5$? What is the corresponding interval? Describe the relationship between these sets.

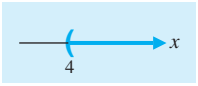
EXAMPLE 6 Solving a Linear Inequality Solve and graph:

$$2(2x + 3) < 6(x - 2) + 10$$

SOLUTION

$$\begin{aligned} 2(2x + 3) &< 6(x - 2) + 10 && \text{Remove parentheses.} \\ 4x + 6 &< 6x - 12 + 10 && \text{Combine like terms.} \\ 4x + 6 &< 6x - 2 && \text{Subtract } 6x \text{ from both sides.} \\ -2x + 6 &< -2 && \text{Subtract 6 from both sides.} \\ -2x &< -8 && \text{Divide both sides by } -2 \text{ and reverse the sense of the inequality.} \end{aligned}$$

$$x > 4 \quad \text{or} \quad (4, \infty)$$



Notice that in the graph of $x > 4$, we use a parenthesis through 4, since the point 4 is not included in the graph.

Matched Problem 6 Solve and graph: $3(x - 1) \leq 5(x + 2) - 5$

EXAMPLE 7 Solving a Double Inequality Solve and graph: $-3 < 2x + 3 \leq 9$

SOLUTION We are looking for all numbers x such that $2x + 3$ is between -3 and 9, including 9 but not -3. We proceed as before except that we try to isolate x in the middle:

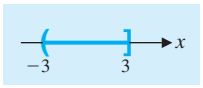
$$-3 < 2x + 3 \leq 9$$

$$-3 - \mathbf{3} < 2x + 3 - \mathbf{3} \leq 9 - \mathbf{3}$$

$$-6 < 2x \leq 6$$

$$\frac{-6}{\mathbf{2}} < \frac{2x}{\mathbf{2}} \leq \frac{6}{\mathbf{2}}$$

$$-3 < x \leq 3 \quad \text{or} \quad (-3, 3]$$



Matched Problem 7 Solve and graph: $-8 \leq 3x - 5 < 7$

Note that a linear equation usually has exactly one solution, while a linear inequality usually has infinitely many solutions.

Applications

To realize the full potential of algebra, we must be able to translate real-world problems into mathematics. In short, we must be able to do word problems.

Here are some suggestions that will help you get started:

PROCEDURE For Solving Word Problems

1. Read the problem carefully and introduce a variable to represent an unknown quantity in the problem. Often the question asked in a problem will indicate the unknown quantity that should be represented by a variable.
2. Identify other quantities in the problem (known or unknown), and whenever possible, express unknown quantities in terms of the variable you introduced in Step 1.
3. Write a verbal statement using the conditions stated in the problem and then write an equivalent mathematical statement (equation or inequality).
4. Solve the equation or inequality and answer the questions posed in the problem.
5. Check the solution(s) in the original problem.

EXAMPLE 8 Purchase Price Alex purchases a plasma TV, pays 7% state sales tax, and is charged \$65 for delivery. If Alex's total cost is \$1,668.93, what was the purchase price of the TV?

SOLUTION

Step 1 **Introduce a variable for the unknown quantity.** After reading the problem, we decide to let x represent the purchase price of the TV.

Step 2 **Identify quantities in the problem.**

Delivery charge: \$65

Sales tax: $0.07x$

Total cost: \$1,668.93

Step 3 **Write a verbal statement and an equation.**

Price	+	Delivery Charge	+	Sales Tax	=	Total Cost
x	+	65	+	$0.07x$	=	1,668.93

Step 4 **Solve the equation and answer the question.**

$$\begin{aligned} x + 65 + 0.07x &= 1{,}668.93 && \text{Combine like terms.} \\ 1.07x + 65 &= 1{,}668.93 && \text{Subtract 65 from both sides.} \\ 1.07x &= 1{,}603.93 && \text{Divide both sides by 1.07.} \\ x &= 1{,}499 \end{aligned}$$

The price of the TV is \$1,499.

Step 5 **Check the answer in the original problem.**

$$\begin{aligned} \text{Price} &= \$1{,}499.00 \\ \text{Delivery charge} &= \$\quad 65.00 \\ \text{Tax} = 0.07 \cdot 1{,}499 &= \$\quad 104.93 \\ \text{Total} &= \$1{,}668.93 \end{aligned}$$

Matched Problem 8 Mary paid 8.5% sales tax and a \$190 title and license fee when she bought a new car for a total of \$28,400. What is the purchase price of the car?

The next example involves the important concept of **break-even analysis**, which is encountered in several places in this text. Any manufacturing company has **costs**, C, and **revenues**, R. The company will have a **loss** if $R < C$, will **break even** if $R = C$, and will have a **profit** if $R > C$. Costs involve **fixed costs**, such as plant overhead, product design, setup, and promotion, and **variable costs**, which are dependent on the number of items produced at a certain cost per item.

EXAMPLE 9 Break-Even Analysis A multimedia company produces DVDs. Onetime fixed costs for a particular DVD are \$48,000, which include costs such as filming, editing, and promotion. Variable costs amount to \$12.40 per DVD and include manufacturing, packaging, and distribution costs for each DVD actually sold to a retailer. The DVD is sold to retail outlets at \$17.40 each. How many DVDs must be manufactured and sold in order for the company to break even?

SOLUTION

Step 1 Let x = number of DVDs manufactured and sold.

Step 2

$$\begin{aligned} C &= \text{cost of producing } x \text{ DVDs} \\ R &= \text{revenue (return) on sales of } x \text{ DVDs} \\ \text{Fixed costs} &= \$48{,}000 \\ \text{Variable costs} &= \$12.40x \\ C &= \text{Fixed costs} + \text{variable costs} \\ &= \$48{,}000 + \$12.40x \\ R &= \$17.40x \end{aligned}$$

Step 3 The company breaks even if $R = C$; that is, if

$$\$17.40x = \$48{,}000 + \$12.40x$$

Step 4

$$\begin{aligned} 17.4x &= 48{,}000 + 12.4x && \text{Subtract } 12.4x \text{ from both sides.} \\ 5x &= 48{,}000 && \text{Divide both sides by 5.} \\ x &= 9{,}600 \end{aligned}$$

The company must make and sell 9,600 DVDs to break even.

Step 5 Check:

Costs	**Revenue**
$48{,}000 + 12.4(9{,}600)$	$17.4(9{,}600)$
$= \$167{,}040$	$= \$167{,}040$

Matched Problem 9 How many DVDs would a multimedia company have to make and sell to break even if the fixed costs are \$36,000, variable costs are \$10.40 per DVD, and the DVDs are sold to retailers for \$15.20 each?

Table 2 **CPI (1982–1984 = 100)**

Year	Index
1960	29.6
1973	44.4
1986	109.6
1999	156.9
2012	229.6

EXAMPLE 10 Consumer Price Index The Consumer Price Index (CPI) is a measure of the average change in prices over time from a designated reference period, which equals 100. The index is based on prices of basic consumer goods and services. Table 2 lists the CPI for several years from 1960 to 2012. What net annual salary in 2012 would have the same purchasing power as a net annual salary of \$13,000 in 1960? Compute the answer to the nearest dollar. (*Source*: U.S. Bureau of Labor Statistics)

SOLUTION

Step 1 Let x = the purchasing power of an annual salary in 2012.

Step 2 Annual salary in 1960 = \$13,000

CPI in 1960 = 29.6

CPI in 2012 = 229.6

Step 3 The ratio of a salary in 2012 to a salary in 1960 is the same as the ratio of the CPI in 2012 to the CPI in 1960.

$$\frac{x}{13{,}000} = \frac{229.6}{29.6}$$ Multiply both sides by 13,000.

Step 4
$$x = 13{,}000 \cdot \frac{229.6}{29.6}$$
$$= \$100{,}838 \text{ per year}$$

Step 5 To check the answer, we confirm that the salary ratio agrees with the CPI ratio:

Salary Ratio	CPI Ratio
$\frac{100{,}838}{13{,}000} = 7.757$	$\frac{229.6}{29.6} = 7.757$

Matched Problem 10 What net annual salary in 1973 would have had the same purchasing power as a net annual salary of \$100,000 in 2012? Compute the answer to the nearest dollar.

Exercises 1.1

Solve Problems 1–6.

1. $2m + 9 = 5m - 6$

2. $3y - 4 = 6y - 19$

3. $2x + 3 < -4$

4. $5x + 2 > 1$

5. $-3x \geq -12$

6. $-4x \leq 8$

Solve Problems 7–10 and graph.

7. $-4x - 7 > 5$

8. $-2x + 8 < 4$

9. $2 \leq x + 3 \leq 5$

10. $-4 < 2y - 3 < 9$

Solve Problems 11–24.

11. $\frac{x}{4} + \frac{1}{2} = \frac{1}{8}$

12. $\frac{m}{3} - 4 = \frac{2}{3}$

13. $\frac{y}{-5} > \frac{3}{2}$

14. $\frac{x}{-4} < \frac{5}{6}$

15. $2u + 4 = 5u + 1 - 7u$

16. $-3y + 9 + y = 13 - 8y$

17. $10x + 25(x - 3) = 275$

18. $-3(4 - x) = 5 - (x + 1)$

19. $3 - y \leq 4(y - 3)$

20. $x - 2 \geq 2(x - 5)$

21. $\frac{x}{5} - \frac{x}{6} = \frac{6}{5}$

22. $\frac{y}{4} - \frac{y}{3} = \frac{1}{2}$

23. $\frac{m}{5} - 3 < \frac{3}{5} - \frac{m}{2}$

24. $\frac{u}{2} - \frac{2}{3} < \frac{u}{3} + 2$

Solve Problems 25–28 and graph.

25. $2 \leq 3x - 7 < 14$

26. $-4 \leq 5x + 6 < 21$

27. $-4 \leq \frac{9}{5}C + 32 \leq 68$

28. $-1 \leq \frac{2}{3}t + 5 \leq 11$

Solve Problems 29–34 for the indicated variable.

29. $3x - 4y = 12$; for y

30. $y = -\frac{2}{3}x + 8$; for x